JGSP **10** (2007) 41–49

JOURNAL OF
Geometry and Symmetry in Physics

NONCOMMUTATIVE GRASSMANNIAN $U(1)$ SIGMA-MODEL AND BARGMANN-FOCK SPACE

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Communicated by Martin Schlichenmaier

Abstract. We consider the Grassmannian version of the noncommutative $U(1)$ sigma-model, which is given by the energy functional $E(P) = \|[a, P]\|^2_{HS}$, where P is an orthogonal projection on a Hilbert space H and the operator $a : H \to H$ is the standard annihilation operator. Using realization of H as the Bargmann-Fock space, we describe all solutions with one-dimensional image and prove that the operator $[a, P]$ is densely defined on H for some class of projections P with infinite-dimensional image and kernel.

1. Introduction

We consider the Grassmannian noncommutative $U(1)$ sigma-model, which is the noncommutative analogue of the classical $\mathbb{C}$-one-dimensional Grassmannian sigma-model. Firstly we describe the latter one. By $\mathrm{Gr}_k(\mathbb{C}^n)$ denote the complex Grassmannian (i.e., the manifold of k-dimensional complex planes in $\mathbb{C}^n$). We will consider its points as orthogonal projections on $\mathbb{C}^n$ with k-dimensional image (and $(n-k)$-dimensional kernel). Then the energy of any map $f : \mathbb{C}P^1 \to \mathrm{Gr}_k(\mathbb{C}^n)$ (i.e., for every z, $f(z)$ is a matrix of k-dimensional orthogonal projection on $\mathbb{C}^n$) is

$$E(f) := \int\limits_{\mathbb{C}P^1} \|\partial_{\bar z} f\|^2_{HS} \mathrm{d}x\mathrm{d}y = \int\limits_{\mathbb{C}P^1} \mathrm{tr}\,(\partial_{\bar z} f)^* \,\partial_{\bar z} f \mathrm{d}x\mathrm{d}y. \qquad (1)$$

Extremals of $E(f)$ (solutions of this model) are called harmonic maps. (For details see [7].)

Under the studying of static $D0$-branes in $D2$-branes (see [3]) there appears the noncommutative analogue of the model above. (This analogue is also considered in [5] and [2].) To describe it, we regard the noncommutative plane $\mathbb{R}^2_\theta$. The transfer is based on the rules of the Weyl calculus of pseudodifferential operators