BOOK REVIEW

Contact Geometry and Non-linear Differential Equations, by Aleksei Kushner, Valentin Lychagin and Vladimir Rubtsov, Cambridge University Press, Cambridge, 2007, xxi + 496pp, ISBN - 978-0-521-82476-7.

Modern mathematical physics could hardly be imagined without making substantial use of modern differential geometry. Writing down and solving appropriate differential equations which are meant to describe the spatial structure and time-evolution of some natural (physical, chemical, biological, etc.) system requires nowadays deep knowledge of most of the algebraic, topological and differential structures that have been developed during the last century and also those being under development now. It would be wrong to consider Mathematics as something abstract and away from reality; on the contrary, its basic concepts and methods of consideration and reasoning follow the appearances and interactions of the real physical world where we live. In fact, any real physical system has spatial structure, it shows definite stability properties, so, it can support its existence and compensate in definite degree the external disturbances through appropriate shape changes and kinematical behavior as a whole without losing identity. Shortly speaking, its time existence is a dynamical process being strongly connected with various and continuous internal and external energy-momentum exchange processes. All the knowledge we have about its existence is clarified and made appropriate to be used for predictions by finding and appropriately combine corresponding part of mathematical structures. As we know, a basic feature of mathematics is to introduce consistency among the structures used. This consistency of mathematical structures models the harmony among the main substructures, interactions among them and local and integral behavior that a given physical system shows. In my view the book under review is one of the best I know in this direction. Let us follow briefly its contents.

It consists of 22 Chapters combined in 5 Parts.

PART I consists of 3 chapters and 17 sections. Chapter 1 contains all necessary information about smooth distributions on manifolds: morphisms, integrability,

symmetries - characteristic and shuffling, curvature of a distribution, appropriately adapted for smooth manifolds Maurer-Cartan equations and the Lie-Bianchi theorem. There are many illustrative examples. Chapter 2 deals mainly with symmetries of ordinary differential equations and related problems and ends with consideration of Schröedinger operators and the related spectral properties of KdV potentials. Chapter 3 considers reduction problems with accent on the algebraic aspects. Three series of model equations are considered. It ends with formulation of the Lie superposition principle and almost-product and almost-complex structures.

PART II consists of 5 chapters. The main subject here is the symplectic algebra (bases, symplectic transformations, lagrangian subspaces, pfaffians, normal forms etc.), considered from the point of view to apply to algebraic properties of nonlinear PDE. An appropriate classification of two-forms on a two-dimensional symplecttic vector is presented, and a classification of three-forms on a six-dimensional symplectic vector is given.

PART III consists of 6 chapters. The first two are devoted to symplectic manifolds (standard structure on cotangent bundles, Koehler manifolds, vector fields and brackets, important submanifolds), contact manifolds (contact structures, contact transformations, Darboux theorem, local description). The next two chapters introduce and consider the Monge-Ampere operators and equations, their symmetries and other properties; good worked out examples are given. Chapter 5 (i.e.,13) of this part considers corresponding conservation laws and discusses also the corresponding variational aspects. The final two chapters of this part consider in a more detail the geometric structures (nonholonomic mainly, Cauchy problem) associated with Monge-Ampere equations on two-dimensional manifolds, and systems of first-order PDE on two-manifolds (mainly Jacobi equations).

PART IV is called "APPLICATIONS", so its contents is concentrated on problems met in applied sciences: acoustics, thermal conductivity, meteorology. The whole spectrum of the developed geometric power is used quite appropriately.

PART V has more theoretical contents, it deals with symplectic and contact classifications of Monge-Ampere equations on two and three dimensional manifolds. The final chapter makes use mainly of jets language (jets of submanifolds, prolongations, symplectic equivalence).

As a whole, (together with the many and very clearly worked out examples presented, which is one of the most important and highly appreciated merits of this book) the text is well written, very well organized and the exposition is very clear. So, I would allow myself to recommend it as a very useful stand-by introduction

to the geometric view on linear and nonlinear differential equations.

I truly believe that reading this book will bring a real pleasure to all young mathematicians and mathematically inclined young physicists having the necessary mathematical luggage in their minds and corresponding adjustment and interests. Moreover, I am fully convinced that getting appropriately acquainted with its contents will bring closely the reader to feel the real correspondence between the real harmony in the physical world and the remarkable mathematical harmony among geometrical structures, and more generally, the correspondence between vital interaction among physical subsystems of a larger physical system and appropriate consistency among mathematical structures living in the same manifold.

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