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DYNAMICAL SYSTEMS TECHNIQUES IN COSMOLOGY. AN EXAMPLE: LQC AND THE EINSTEIN STATIC UNIVERSE

LUCA PARISI

Communicated by Akira Yoshioka

Abstract. Dynamical Systems theory is a valuable tool in the profound study of Cosmology, as qualitative methods allow to characterize cosmological solutions on the basis of their relevant physical features (e.g. stability and asymptotic behaviour). Here we briefly review some well established results for cosmological models in the framework of General Relativity. Then we consider a family of modified cosmological models which are gaining more and more relevance, namely Loop Quantum Cosmologies. In particular we analyze the geometrical structure and dynamical properties of the model presented in earlier.

1. Introduction

We consider the application of dynamical systems theory to mathematical models from classical and semiclassical Cosmology. In particular, the Einstein Static (ES) universe in General Relativity (GR) is reviewed along with its stability properties. Then static solutions of the so called semiclassical Loop Quantum Cosmology (LQC) modified equations for homogeneous and isotropic closed cosmological models (K = 1) with a cosmological constant Λ are considered.

The paper is structured as follows. In the first section we review some basic notions about Cosmology which will be helpful for the following discussion. In the second section we introduce some standard definitions from Dynamical Systems theory which are employed, together with numerical integrations, in the following sections. In the third section, we describe the ES model in GR and the dynamical system approach shows that the ES solution is unstable to homogeneous perturbations. Then we focus on Loop Quantum Gravity [17] techniques that, applied to Cosmology, give rise to LQC [4]. LQC strongly modifies the high-energy dynamics of Friedman-Robertson-Walker models. One of its more remarkable features is that it removes the big-bang singularity [15]. We show that LQC modifications can lead to an ES model which is neutrally stable for a large enough positive value of Λ.