BOOK REVIEW

Geometric Mechanics and Symmetry. From Finite to Infinite Dimensions, by Darryl D. Holm, Tanya Schmah and Cristina Stoica, Oxford University Press, Oxford, 2009, xi + 515 pp., ISBN: 978-0-19-921290-3

The purpose of the book is to provide the unifying viewpoint of Lagrangian and Hamiltonian mechanics in the coordinate-free language of differential geometry in the spirit of the Marsden-Ratiu school. The book is similar in content - although less formal - to the book by J. Marsden and T. Ratiu [7]. One can also mention the companion two-volumes book by Holm [4, 5] written at a more basic level, and that one can recommend as an introductory reading. The classical treatises on the subject are the books by Abraham-Marsden [1], Arnold [2] and Libermann-Marle [6].

Typical applications are N-particle systems, rigid bodies, continua such as fluids and electromagnetic systems that illustrate the powerfulness of the adopted point of view. The geometrical structure allows the covering of both the finite-dimensional conservative case (first part of the book) and the infinite dimensional situation in the second part. The notion of symmetry here is central, as it allows a reduction of the number of dimensions of the mechanical systems, and further exploits the conserved quantities (momentum map) associated to symmetry. Lie group symmetries, Poisson reduction and momentum maps are first discussed.

The concepts are introduced in a progressive and clear manner in the first part of the book. The second part devoted to infinite dimensional systems is motivated by the identification of Euler's ideal fluid motion with the geodesic flow on the group of volume-preserving diffeomorphism. The Euler-Poincaré (EP) variational principle for the Euler fluid equations is exposed in the framework of geometric mechanics, in association with Lie-Poisson Hamiltonian structure of Noether's theorem and momentum maps. Original applications of the Euler-Poincaré equations to solitons, computational anatomy, image matching, or geophysical fluid dynamics are given at the end of the second part of the book.

Here the first chapter recapitulates the Newtonian, Lagrangian and Hamiltonian

mechanics of N-particle systems in Euclidean space. The prerequisite for the extension of the Lagrangian and Hamiltonian mechanics to systems living on manifolds is exposed in Chapter 2 and Chapter 3. In addition to those topics, Chapter 4 ends up with a brief presentation of the concepts of symmetry, reduction and conservation laws, as articulated in Noether's theorem. Lie groups and their action together with Lie algebra are exposed in Chapter 5 and Chapter 6, as a prerequisite to analysing mechanical systems having their configuration space as a Lie group. Chapter 6 ends with the notion of Poisson's reduction. Chapter 7 proceeds with the Euler-Poincaré reduction, which is the specific procedure for obtaining the reduced dynamics on the quotient space, illustrated in the case of the rigid body and the heavy top (rigid body rotating with a fixed point of support in a constant gravitational field). The Lie group symmetries of the Hamiltonian H generate conserved quantities, a notion being encapsulated in the concept of a momentum map ($\mathfrak{g}$ is the Lie algebra of the Lie group G leaving H invariant) that is conserved under the flow of the Hamiltonian vector field (Chapter 8). The Hamiltonian counterpart of Euler-Poincaré reduction is the Lie-Poisson reduction, covered in Chapter 9. The concepts and results of previous chapters are applied in Chapter 10 to the affine rigid body motion.

The second part of the book relies on a pioneering contribution by Arnold [3], who observed that the motion of the Euler ideal fluid may be identified with the geodesic flow on the group of volume-preserving smooth diffeomorphism of the region occupied by the fluid. This geometrical set up of ideal continuum fluid motion is exposed in Chapter 11, which ends with the EPDiff equation for the covector function, adopting the norm of the kinetic energy. The singular solutions of the EPDiff equation corresponding to peakon travelling waves are examined in Chapter 12. Integrability of EPDiff in 1D and soliton solutions are topics discussed in Chapter 13. Chapter 14 discusses the n-dimensional generalization of the 1D singular peakon solutions, providing numerical simulations of nonlinear wave reconnection in 2D. Applications of EPDiff to contour matching for the shape analysis of deformable biological structures is the object of Chapter 15, thereby providing an interesting and original application of EPDiff to the dynamics of morphology. Pursuing in the same topic, Chapter 16 explains the geometrical theory of image matching and its relation to the Euler-Poincaré equations using the metamorphosis approach. The EP theorem for ideal continuum flows with advected quantities is discussed in Chapter 17. The book concludes the applications of EP theorem in Chapter 18 with consideration of geophysical fluid dynamics, which is of importance in modelling the Earth climate changes. The Euler-Poincaré approach brings some explanations in the conservation properties and circulation dynamics

of the obtained nonlinear and nonlocal equations.

The exposition is quite clear, as the concepts are systematically illustrated by well chosen examples, and exercises (selected exercises are corrected) are provided for a good assimilation of ideas and methods. The required prerequisites are solid undergraduate courses in linear algebra, multivariate calculus, standard methods for solving ODE's and PDE's, and possibly variational principles.

In conclusion, the book provide a very comprehensive presentation of ideas and methods from geometric mechanics, aimed at the graduate student level, but it could also be of interest for specialists who want to refresh their knowledge in this modern, elegant and unifying formulation of Lagrangian and Hamiltonian mechanics.

References

[1] Abraham R. and Marsden J., *Foundations of Mechanics*, 2nd ed., Addison-Wesley, Massachusetts, 1978.

[2] Arnold V., *Mathematical Methods of Classical Mechanics*, 2nd ed., Springer, Berlin, 1978.

[3] Arnold V., *Sur la géometrie differentialle des groupes de Lie de dimension infinie et se application à l'hydrodynamique des fluids parfaits*, Ann. Inst. Fourier **16** (1966) 319–361.

[4] Holm D., *Geometric Mechanics. Part I*: *Dynamics and Symmetry*, Imperial College Press, London, 2008.

[5] Holm D., *Geometric Mechanics. Part II*: *Rotating, Translating and Rolling*, Imperial College Press, London, 2008.

[6] Libermann P. and Marle C.-M., *Symplectic Geometry and Analytical Mechanics*, Reidel, Dordrecht, 1987

[7] Marsden J. and Ratiu T., *Introduction to Mechanics and Symmetry*: *A Basic Exposition of Classical Mechanical Systems*, Texts in Applied Mathematics, Springer, New York, 2002.

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