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GLOBAL WORLD FUNCTIONS

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Abstract. Starting from the Amann-Conley-Zehnder finite reduction framework in the non-compact Viterbo's version we discuss the existence of global generating function with a finite number of auxiliary parameters describing the two-points Characteristic Relation related to the geodesic problem in the Hamiltonian formalism. This applies both to Analytical Mechanics and to General Relativity – we construct a global object generalizing the World Function introduced by Synge, which is well-defined only locally. Whenever the auxiliary parameters can be fully removed, Synge's World Function is restored.

1. Introduction

In the textbook by J. Synge [19] one can find the following definition:
Let $P'(x')$ and $P(x)$ be two points in the space-time, joined by a geodesic Γ with equations $x^i = \xi^i(u)$ where u is a special[1] *parameter. Then the integral*

$$\Omega(P'P) = \Omega(x',x) = \frac{1}{2}(u_1 - u_0)\int_{u_0}^{u_1} g_{ij}U^iU^j \,\mathrm{d}u$$

taken along Γ with $U^i = \mathrm{d}\xi^i/\mathrm{d}u$, has a value independent of the particular special parameter chosen. If, as we shall suppose, the points P' and P determine a unique geodesic passing through them, then Ω is a function of these two points. As a function of the eight variables x' and x we shall call it the world-function *of space-time.*

The World Function had a rather troubled history. The main criticism is that it has only a local meaning, and even in simple cases we cannot use it for global analysis. Really, Synge recognized this limitation a few lines after its definition.

[1] special parameters are the representative elements of a class of parameters invariant by affine transformations.