# GEOMETRIC INTERPRETATION OF THE RECURSION OPERATORS FOR THE GENERALIZED ZAKHAROV-SHABAT SYSTEM IN POLE GAUGE ON THE LIE ALGEBRA $A_2$

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**Abstract.** We consider the recursion operator approach to the soliton equations related to a generalized Zakharov-Shabat auxiliary linear system in pole gauge on the Lie algebra $A_2 = \mathfrak{sl}(3, \mathbb{C})$ and show that the recursion operator can be identified with the dual to a Nijenhuis tensor for a Poisson-Nijenhuis structure on the manifold of potentials.

## 1. Introduction

The soliton equations or completely integrable equations have been object of intense study since their discovery. Their most essential property is that they admit a Lax representation $[L, A] = 0$. In it $L, A$ are linear operators on $\partial_x, \partial_t$ depending also on some functions $q_i(x, t), 1 \leq i \leq s$ ('potentials') and a spectral parameter $\lambda$. The equation $[L, A] = 0$ should be satisfied identically in $\lambda$ and in this way the Lax equation $[L, A] = 0$ is equivalent to a system of partial differential equations for $q_i(x, t)$. Usually one fixes the linear problem $L\psi = 0$ (auxiliary linear problem) and considers all the evolution equations (of certain form of course) one can obtain changing the operator $A$. These equations are called nonlinear evolution equations (NLEEs) associated (related) with $L$ (or with the linear system $L\psi = 0$). There are several different schemes to resolve them but the essential point is that the Lax representation permits to pass from the original evolution defined by the equation to the evolution of some spectral data related to the problem $L\psi = 0$ which is linear and consequently easily found. From this data the potentials can be recovered by a process called Inverse Scattering Method, see the monographs [5, 8].

The Generalized Zakharov-Shabat (GZS) system presented below is a paradigm of auxiliary linear problem. It can be written as follows

$$L\psi = (\mathrm{i}\partial_x + q(x) - \lambda J)\, \psi = 0. \tag{1}$$

Here $q(x)$ and $J$ belong to some fixed simple Lie algebra $\mathfrak{g}$ in some finite dimensional irreducible representation. The element $J$ is regular, that is the kernel of $\mathrm{ad}_J$