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CLASSICAL-QUANTUM CORRESPONDENCE AND WAVE PACKET SOLUTIONS OF THE DIRAC EQUATION IN A CURVED SPACE-TIME

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Abstract. The idea of wave mechanics leads naturally to assume the well-known relation $E = \hbar\omega$ in the specific form $H = \hbar W$, where H is the classical Hamiltonian of a particle and W is the dispersion relation of the sought-for wave equation. We derive the expression of H in a curved space-time with an electromagnetic field. Then we derive the Dirac equation from factorizing the polynomial dispersion equation corresponding with H. Conversely, summarizing a recent work, we implement the geometrical optics approximation into a canonical form of the Dirac Lagrangian. Euler-Lagrange equations are thus obtained for the amplitude and phase of the wave function. From them, one is led to define a four-velocity field which obeys exactly the classical equation of motion. The complete de Broglie relations are then derived as exact equations.

1. Introduction

1.1. Context of This Work

The long-standing problem of quantum gravity may mean, of course, that we should try to better understand gravity and the quantum. More specifically, it may mean that we should try to better understand the transition between the classical and the quantum, especially in a curved space-time. Quantum effects in the classical gravitational field are indeed being observed on neutral particles such as neutrons [11, 15, 19] or atoms [13, 18], with the neutrons being spin $\frac{1}{2}$ particles. This together motivates investigating the "classical-quantum correspondence"—the correspondence between a classical Hamiltonian and a quantum wave equation—for the Dirac equation in a curved space-time.