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ON HENRI POINCARÉ'S NOTE "Sur une forme nouvelle des équations de la Mécanique"

CHARLES-MICHEL MARLE *

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Abstract. We present in modern language the contents of the famous note published by Henri Poincaré in 1901, in which he proves that, when a Lie algebra acts locally transitively on the configuration space of a Lagrangian mechanical system, the well known Euler-Lagrange equations are equivalent to a new system of differential equations defined on the product of the configuration space with the Lie algebra. We write these equations, called the *Euler-Poincaré equations*, in an intrinsic form, without any reference to a particular system of local coordinates, and prove that they can be conveniently expressed in terms of the Legendre and momentum maps. We discuss the use of the Euler-Poincaré equation for reduction (a procedure sometimes called *Lagrangian reduction* by modern authors), and compare this procedure with the well known Hamiltonian reduction procedure (formulated in modern terms in 1974 by J. Marsden and A. Weinstein). We explain how a break of the symmetry in the phase space produces the appearance of a semi-direct product of Lie groups.

Contents

[0*] In memory of Jean-Marie Souriau, founder of the modern theory of Geometrical Mechanics, with respect and admiration.

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