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SIGMA FUNCTIONS FOR A SPACE CURVE OF TYPE $(3, 4, 5)$

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Abstract. In this article, a generalized Kleinian sigma function for an affine $(3, 4, 5)$ space curve of genus two was constructed as the simplest example of the sigma function for an affine space curve, and in terms of the sigma function, the Jacobi inversion formulae for the curve are obtained. An interesting relation between a space curve with a semigroup generated by $(6, 13, 14, 15, 16)$ and Norton number associated with Monster group is also mentioned in the Appendix by the second author.

1. Introduction

Recently the Kleinian sigma function for hyperelliptic curves, a natural generalization of the Weierstrass sigma function, is re-evaluated because in terms of the sigma functions, it is more convenient to investigate the properties of the abelian functions and their interesting properties are revealed naturally [2, 6, 17].

Further Enolskii, Eilbeck, and Leykin [5] discovered a construction which generalizes the Kleinian sigma function associated with hyperelliptic curves to one for an affine (r, s) plane curve, where r and s $(r < s)$ are coprime positive integers $g = (r-1)(s-1)/2$. There they have constructed also the fundamental differential of the second kind over an affine (r, s) plane curve and using it, obtained the Legendre relation as the symplectic structure over the curve. Using the Legendre relation, they defined the generalized Kleinian sigma function over the image of the abelian map $\mathbb{C}^g$. They have found also the natural Jacobi inversion formulae in terms of their sigma function. We call this construction *EEL construction* in this article. Using the EEL construction, we have some interesting results [20, 21].

In this article, we consider a generalized Kleinian sigma function for an affine $(3, 4, 5)$ space curve of genus two, which is the simplest affine space curve. Our purpose of this article is to show that the sigma function is also defined for an affine space curve as we can do for plane curves.

Following the EEL-construction, we define the fundamental differential of the second kind over it and obtain the Legendre relation as the symplectic structure over it.

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