JGSP **31** (2013) 105–117

JOURNAL OF
Geometry and Symmetry in Physics
ISSN 1312-5192

# CLASSICAL AND QUANTUM SYMMETRIES REDUCTION AND INTEGRABILITY

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Communicated by Gijs Tuynman

**Abstract.** Completely integrable systems always admit more alternative Hamiltonian descriptions. The geometrical formulation of quantum systems shows that similar conclusions hold true also for quantum systems. In addition, the description of quantum systems on Hilbert manifolds, e.g., the complex projective space, shows that not only quantum systems admit alternative Hamiltonian descriptions, they also admit alternative linear descriptions.

## 1. Introduction

In his *Lectures on Dynamics* [7], Jacobi starts with the problem of integrating the differential equations of motion. He explicitly says: In *Mécanique Analytique one finds everything related to the problem of setting up and transforming the differential equations, but very little on their integration.*

He goes on to elaborate what we nowdays call the Hamilton-Jacobi theory and elaborates on constants of the motion and symmetries.

The aim of our paper is to present a more general point of view in which the Hamilton-Jacobi theory is only an instance of the general procedure of integrating a system by reducing it to a *normal form*. In this respect we follow the view point of Birkhoff, all dynamical systems in the same orbit of the diffeomorphism group enjoy the same properties, therefore to study the integration problem one may select a particular representative of the equivalence class and consider it as a "normal form".

In this picture, the Hamilton-Jacobi procedure becomes a way to reduce a given Hamiltonian system to a normal form by replacing the full diffeomorphism group with the subgroup of canonical transformation necessary to achieve the transition to the normal form, the particular trnsformation is found by means of a generating function, solution of the Hamilton-Jacobi equation associated with the starting Hamiltonian functions, the comparison Hamiltonian and the one we want to tranform.

doi: 10.7546/jgsp-31-2013-105-117