



# APPLICATIONS OF LUSTERNIK-SCHNIRELMANN CATEGORY AND ITS GENERALIZATIONS

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**Abstract.** This paper explores some applications of Lusternik-Schnirelmann theory and its recent offshoots. In particular, we show how the LS category of real projective space leads to the Borsuk-Ulam theorem and the Brouwer fixed point theorem. After the development of some LS categorical tools, we also show the importance of LS category in understanding the Arnold conjecture on fixed points of Hamiltonian diffeomorphisms. We then examine ways in which LS category fits into the framework of differential geometry. In particular, we give a refinement of Bochner’s theorem on the first Betti number of a non-negatively Ricci-curved space and a Bochner-like corollary to a recent theorem of Kapovitch-Petrulin-Tuschmann. Finally, we survey the new LS categorical notion of topological complexity and its relation to the motion planning problem in robotics.

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