



BOOK REVIEW

Differential Geometry From a Singularity Theory Viewpoint, by Shyuichi Izumiya, Maria del Carmen Romero Fuster, Maria Aparecida Soares Ruas and Farid Tari, World Scientific, Singapore 2016, xiii+368 pp, ISBN 978-9814590440

It is well known that during the 18th century and the 19th century, differential geometry has arose and developed as a result of and in fascinating connection to the mathematical analysis of curves and surfaces. Since then, many works have been devoted to the general theory of curves and surfaces in an Euclidean space (or more generally in a Riemannian manifold). So now, we have an extensive knowledge about their local geometry as well as of their global geometry [1]. Also, there were many important links between geometry of curves and surfaces and other sciences especially physics, biology, chemistry, engineering, etc... Besides the works on geometry of curves and surfaces in Euclidean space, there are many important works on these subjects in affine, hyperbolic or Minkowski spaces. As we know, Minkowski spaces were introduced by the mathematician Hermann Minkowski for Maxwell's equations of electromagnetism, the mathematical structure of Minkowski space-time was shown to be an immediate consequence of the postulates in special relativity. Minkowski space is also closely associated with Einstein's theory of special relativity, and is the most common mathematical structure on which special relativity is formulated. The theory of curves and surfaces are important to understand the special relativity and its tools. For example, the geometry of null hypersurfaces in space-time has played an important role in the development of general relativity, as well as in mathematics and physics of gravitation. It is necessary, for example, for understanding the causal structure of space-time, black holes, asymptotically flat systems and gravitational waves [2].

This book is a welcome addition to the literature on differential geometry in Euclidean and Minkowski spaces. This book offers a new look at the classical subjects of curves and surfaces, namely from the point of view of the singularity theory. The core of the book are the research results and interests of the authors and their collaborators.

The book is organized as follows: Chapter 1 - The Case for the Singularity Theory Approach. In this chapter, some well-known facts are given for planar curves, space curves and some surfaces in Euclidean space and by using singularity theory some results are given for the extrinsic geometry of submanifolds of the Euclidean spaces.

Chapter 2 - Submanifolds of the Euclidean Space. To understanding the whole book, basic facts about the extrinsic geometry of a submanifold M of dimension n of the Euclidean space \mathbb{R}^{n+r} , with $r \geq 1$ are introduced.

Chapter 3 - Singularities of Germs of Smooth Mappings. After giving some information about historical development of singularity theory, the notion of singularities of smooth mappings is given.

Chapter 4 - Contact Between Submanifolds of \mathbb{R}^n . In this chapter, the authors investigate the concept of contact between submanifolds as a singularity theory. The relation between the contact of equidimensional submanifolds of a given manifold and \mathcal{K} - singularities of maps was introduced by Mather in [3] and the general theory of contact between submanifolds of any dimensions of a given manifold was developed by Montaldi [4].

Chapter 5 - Lagrangian and Legendrian Singularities. In this chapter, firstly some basic concepts in symplectic and contact geometries are given. Then the authors examine the relation between the theory of contact and the theory of Lagrangian and Legendrian singularities.

Chapter 6 - Surfaces in the Euclidean Space. After giving some well-known results for a surface M in Euclidean space \mathbb{R}^3 , the contact of M with planes, lines and spheres are studied and the singularities, respectively the height functions, orthogonal projections and distance squared functions on M are considered.

Chapter 7 - Surfaces in the Euclidean 4-Space. This chapter considers the extrinsic differential geometry of a surface M immersed in \mathbb{R}^4 . A careful study the contact of the surface with flat objects (hyperplanes, planes and lines) and derive from it the extrinsic properties of the surface. This contact is affine invariant, so the derived properties from it are also affine invariant.

Chapter 8 - Surfaces in the Euclidean 5-Space. The principal aim of this chapter is to describe how the singularity techniques can be applied to the analysis of the extrinsic geometry of surface in higher codimensions.

Chapter 9 - Spacelike Surfaces in the Minkowski Space-Time. In the singularity theory, it is well-known that the geometrical properties associated to the contact of spacelike submanifolds with spacelike models do not differ much from those of submanifolds of Euclidean case. However the contact of submanifolds with

lightlike hyperplanes or with lightcones are more interesting and the geometrically rich situation than the Euclidean and spacelike cases.

The book contains many figures, tables and the Bibliography includes many recent books and papers (most of them belong to the authors of this book).

The book covers an active, interesting and fresh research area. It is easy to read and follow the chapters. It will be very useful not only for mathematicians but also for the researchers in other sciences like physicists and applied mathematicians.

References

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