



# GEOMETRY OF TWISTED SASAKI METRIC

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**Abstract.** Let  $(M, g)$  be a  $n$ -dimensional smooth Riemannian manifold. In the present paper, we introduce a new class of natural metrics denoted by  $G^{f,h}$  and called twisted Sasaki metric on the tangent bundle  $TM$ . We studied the geometry of  $(TM, G^{f,h})$  by giving a relationships of the curvatures, Einstein structure, scalar and sectional curvatures between  $(TM, G^{f,h})$  and  $(M, g)$ .

MSC: 58A03, 58A05.

Keywords: Einstein structure, natural metrics, twisted Sasaki metric

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## 1. Preliminaries

We recall some basic facts about the geometry of the tangent bundle. In the present paper, we denote by  $\Gamma(TM)$  the space of all vector fields of a Riemannian manifold  $(M, g)$ . Let  $(M, g)$  be an  $n$ -dimensional Riemannian manifold and  $(TM, \pi, M)$