



## CHARACTERIZATION AND COMPUTATION OF MATRICES OF MAXIMAL TRACE OVER ROTATIONS

JAVIER BERNAL AND JIM LAWRENCE

Communicated by Ivailo M. Mladenov

**Abstract.** Given a  $d \times d$  matrix  $M$ , it is well known that finding a  $d \times d$  rotation matrix  $U$  that maximizes the trace of  $UM$ , i.e., that makes  $UM$  a matrix of maximal trace over rotation matrices, can be achieved with a method based on the computation of the singular value decomposition (SVD) of  $M$ . We characterize  $d \times d$  matrices of maximal trace over rotation matrices in terms of their eigenvalues, and for  $d = 2, 3$ , we identify alternative ways, other than the SVD, of computing  $U$  so that  $UM$  is of maximal trace over rotation matrices.

*MSC:* 15A18, 15A42, 65H17, 65K99, 93B60

*Keywords:* Eigenvalues, orthogonal, Procrustes, rotation, SVD, trace, Wahba

---

### Contents

<b>1</b>	<b>Introduction</b>	<b>22</b>
<b>2</b>	<b>Reformulation of Problems as Maximizations of the Trace of Matrices Over Rotations</b>	<b>24</b>
<b>3</b>	<b>Characterization of Matrices of Maximal Trace Over Rotations</b>	<b>25</b>
<b>4</b>	<b>The Two-Dimensional Case: Computation without SVD</b>	<b>36</b>
<b>5</b>	<b>The Three-Dimensional Case: Computation without SVD</b>	<b>40</b>
<b>6</b>	<b>The Three-Dimensional Case Revisited</b>	<b>46</b>
	<b>References</b>	<b>52</b>
	doi: 10.7546/jgsp-53-2019-21-53	21