

# AN OVERVIEW OF THE HISTORY OF PROJECTIVE REPRESENTATIONS OF GROUPS

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Communicated by Ivailo M. Mladenov

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**Abstract.** An overview of the history of projective representations (= spin representations) of groups, preceded by the prehistory of studies on the theory of quaternion due to Rodrigues and Hamilton. Beginning with Schur, we cover many mathematicians until today, and also physicists Pauli and Dirac.

*MSC:* 01A85, 20C25

*Keywords:* Dirac equation, Lorentz group, Pauli matrices, projective representations and spin characters of groups, spin representations of groups

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