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SMOOTH EXTENSIONS AND SPACES OF SMOOTH AND HOLOMORPHIC MAPPINGS

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Abstract. In this paper we present a new notion of a smooth manifold with corners and relate it to the commonly used concept in the literature. We also introduce complex manifolds with corners show that if M is a compact (respectively, complex) manifold with corners and K is a smooth (respectively, complex) Lie group, then $C^{\infty}(M,K)$ (respectively, $\mathcal{O}(M,K)$) is a smooth (respectively, complex) Lie group.

1. Introduction

We introduce the notion of a smooth manifold with corners, which is an extension of the existing notion of smooth manifolds with corners or boundary for the finite-dimensional case (cf. [6] or [7, Ch. 2]). The notation presented here is the appropriate notion for a treatment of mapping spaces and Whitney's extension theorem [9] implies that for finite-dimensional smooth manifolds our definition coincides with the one given in [6]. We give an alternative proof of a similar statement by elementary methods from real Analysis (cf. also [4, Theorem 22.17] and [4, Proposition 24.10]).

We also introduce complex manifolds with corners and derive several properties of the spaces $C^{\infty}(M,K)$ and $\mathcal{O}(M,K)$. Eventually it turns out that these mapping spaces are smooth (respectively, complex) Lie groups. This is in particular interesting since it seems to be the only way to put a complex or even smooth structure onto spaces of holomorphic mappings since the Open Mapping Theorem implies that in the case of a closed compact manifolds all holomorphic maps are constant. With the results of this paper a Lie theoretic treatment of groups like $\mathcal{O}(M,K)$ for non-compact M becomes possible as a projective limit of Lie groups.