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TWISTOR INTEGRAL REPRESENTATIONS OF SOLUTIONS OF THE SUB-LAPLACIAN

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Abstract. The twistor integral representations of solutions of the Laplacian on the complex space are well-known. The purpose of this article is to generalize the results above to that of the sub-Laplacian on the odd-dimensional complex space with the standard contact structure.

Introduction

The twistor integral representations of solutions of the complex Laplacian on the complex space $\mathbb{C}^{2n}$ of even dimension $2n$ are well-known. We also showed them on $\mathbb{C}^{2n-1}$ of odd dimension $2n-1$ before. The purpose of this article is to generalize the results above to that of the complex sub-Laplacian on $\mathbb{C}^{2n-1}$ with the standard contact structure. The details and further discussion will appear elswhere.

Let (x_i, y_i, z) $i = 1, \ldots, n-1$ be the standard coordinate system of $\mathbb{M} = \mathbb{C}^{2n-1}$. We give $\mathbb{M}$ a contact structure defind by

$$\theta = \mathrm{d}z - \sum_{i=1}^{n-1}(y_i \, \mathrm{d}x_i - x_i \, \mathrm{d}y_i)$$

called a contact form. The contact distribution D on $\mathbb{M}$ is defined by $\theta = 0$. The vector fields

$$X_i = \frac{\partial}{\partial x_i} + y_i \frac{\partial}{\partial z}\,, \quad Y_i = \frac{\partial}{\partial y_i} - x_i \frac{\partial}{\partial z}\,, \quad i = 1, \ldots, n-1$$

furnish a basis of D. Let us join $Z = \dfrac{\partial}{\partial z}$ to them. By $[Y_i, X_i] = 2Z;\ i = 1, \ldots, n-1$ they form a basis of the Heisenberg algebra.