

NAMBU DYNAMICS, n -LIE ALGEBRAS AND INTEGRABILITY

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Abstract. We present a generalized formulation of Poisson dynamics suitable to describe *the n -bodies interactions*. Examples are given of physical systems endowed with such a general structure.

1. Introduction

The Nambu dynamics is an example of n -Poisson structure which is a special n -Lie algebra. The latter was introduced for the first time by V. T. Filippov [4] in 1985 who gave first examples, developed first structural concepts, like simplicity, in this context and classified n -Lie algebras of dimensions $n + 1$ which is parallel to the Bianchi classification of three-dimensional Lie algebras.

Filippov defines an n -Lie algebra structure to be an n -ary multi-linear and anti-symmetric operation

$$[v_1, \dots, v_{n-1}]$$

which satisfies the n -ary Jacobi identity:

$$\begin{aligned} & [v_1, \dots, v_{n-1}, [u_1, \dots, u_n]] \\ &= \sum_{i=1}^n [u_1, \dots, u_{i-1}, [v_1, \dots, v_{n-1}, u_i], u_{i+1}, \dots, u_n] \quad (1) \end{aligned}$$

Such an operation, realized on the smooth function algebra of a manifold and additionally assumed to be an n -derivation, is an **n -Poisson structure**.

This general concept, however, was not introduced by Filippov. A first proposal goes back to M. L. Albeggiani (1936) [13] who introduced, in a different context,