Twelfth International Conference on
Geometry, Integrability and Quantization
June 4–9, 2010, Varna, Bulgaria
Ivaïlo M. Mladenov, Gaetano Vilasi
and Akira Yoshioka, Editors
Avangard Prima, Sofia 2011, pp 170–177

A FORMULAE FOR THE SPECTRAL PROJECTIONS OF TIME OPERATOR

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Abstract. In this paper, we study the one-level Friedrichs model by using the quantum time super-operator that predicts the excited state decay inside the continuum. Its survival probability decays exponentially in time.

1. Introduction

In this paper we shall study the concept of survival probability of an unstable quantum system introduced in [6] and we shall test it in the Friedrichs model [7]. The survival probability should be a monotonically decreasing time function and this property could not exists in the framework of the usual Weisskopf-Wigner approach [1, 2, 8, 11]. It could only be properly treated if it is defined through an observable T (time operator) whose eigenprojections provide the probability distribution of the time of decay. The equation defining the **time operator** T is

$$U_{-t}TU_t = T + tI \tag{1}$$

where U_t is the unitary group of states evolution. It is known that such an operator cannot exist when the evolution is governed by the Schrödinger equation, since the Hamiltonian has a bounded spectrum from below, and this contradicts the equation

$$[H, T] = \mathrm{i}I \tag{2}$$

in the Hilbert space of pure states $\mathcal{H}$. However, the time operator T can exist under some conditions, for mixed states. They can be embedded [3, 6, 12] in the "Liouville space", denoted $\mathcal{L}$, that is the space of Hilbert-Schmidt operators ρ on $\mathcal{H}$ such that $\mathrm{Tr}(\rho^*\rho) < \infty$, equipped with the scalar product $<\rho, \rho'>= \mathrm{Tr}(\rho^*\rho')$.