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CONSTANT MEAN CURVATURE SURFACES AT THE INTERSECTION OF INTEGRABLE GEOMETRIES

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Abstract. The constant mean curvature surfaces in three-dimensional space-forms are examples of isothermic constrained Willmore surfaces, characterized as the constrained Willmore surfaces in three-space admitting a conserved quantity. Both constrained Willmore spectral deformation and constrained Willmore Bäcklund transformation preserve the existence of a conserved quantity. The class of constant mean curvature surfaces in three-dimensional space-forms lies, in this way, at the intersection of several integrable geometries, with classical transformations of its own, as well as constrained Willmore transformations and transformations as a class of isothermic surfaces. Constrained Willmore transformation is expected to be unifying to this rich transformation theory.

1. Introduction

Minimal surfaces appear as the area-minimizing surfaces amongst all those spanning a given boundary. The Euler-Lagrange equation of the underlying variational problem turns out to be the zero mean curvature equation. A physical model of a minimal surface can be obtained by dipping a wire frame into a soap solution. The resulting soap film is minimal, in the sense that it always tries to organize itself so that its surface area is as small as possible whilst spanning the wire contour. This minimal surface area is reached for the flat position, which is also the position in which the membrane is the most relaxed, i.e., where the elastic energy is minimal - these surfaces are elastic energy extremals and, in this way, examples of Willmore surfaces. In fact, a classical result by Thomsen [23] characterizes isothermic Willmore surfaces in three-space as minimal surfaces in some three-dimensional space-form.