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HARMONIC ANALYSIS ON LAGRANGIAN MANIFOLDS OF INTEGRABLE HAMILTONIAN SYSTEMS

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Abstract. For an integrable Hamiltonian system we construct a representation of the phase space symmetry algebra over the space of functions on a Lagrangian manifold. The representation is a result of the canonical quantization of the integrable system using separating variables. The variables are chosen in such way that half of them parameterizes the Lagrangian manifold, which coincides with the Liouville torus of the integrable system. The obtained representation is indecomposable and non-exponentiated.

1. Introduction

The problem of quantization on a Lagrangian manifold has arisen from the theory of geometric quantization [4]. But the question how to choose a proper Lagrangian manifold remains open. Dealing with a dynamical system we use its Liuoville torus as a Lagrangian manifold. This choice guarantees that the representation space consists of holomorphic functions - functions on the special Lagrangian manifold whose complexification serves as a phase space of the system.

According to the orbit method one can construct an integrable soliton hierarchy (hierarchy of equations of soliton type) on the orbits of a loop group [3]. Finite gap phase spaces for the integrable hierarchy appeared to consist of orbits of finite quotient algebras corresponding to the loop group. On such phase spaces one can introduce canonical separating variables (Darboux coordinates), which represent points of a spectral curve [2]. The curve is hyperelliptic for many interesting integrable systems. A half of the separating variables parametrizes the Lagrangian manifold which is the Liouville torus for the integrable system in question, and the complexified Lagrangian manifold serves as a generalized Jacobian of the spectral curve.