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CYLINDRICAL FLUID MEMBRANES AND THE EVOLUTIONS OF PLANAR CURVES

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Abstract. An interesting relation between the mKdV equation and the cylindrical equilibrium shapes of fluid membranes is observed. In our setup mKdV arises from the study of the evolution of planar curves in the normal direction.

1. Introduction

This paper unifies and extends the results of two articles, and shows a relation between two problems that appears unrelated.

The first problem comes from the study of equilibrium shapes of fluid membranes. One starts with a functional proposed by Helfrich (see [2], [8]) and studies the corresponding Euler-Lagrange equation. The equilibrium shapes are given as the extrema of the functional

$$\mathcal{F} = \frac{k_c}{2}\int_S (2H + \mathbb{h})^2 \mathrm{d}A + k_G \int_S K \mathrm{d}A + \lambda \int_S \mathrm{d}A + p \int \mathrm{d}V. \tag{1}$$

Notice that $\mathcal{F}$ is closely related to the Willmore energy functional. The Euler-Lagrange equation associated with $\mathcal{F}$ is as follows

$$2k_c \Delta_{\mathcal{S}} H + k_c(2H + \mathbb{h})(2H^2 - \mathbb{h}H - 2K) - 2\lambda H + p = 0. \tag{2}$$

Here H and K are the mean and Gauss curvatures respectively, k_c and k_G - bending and Gaussian rigidity constants of the membrane, $\mathbb{h}$ is the **spontaneous curvature** constant, p and λ - the Lagrange multipliers corresponding to the fixed volume and total membrane area and $\Delta_{\mathcal{S}}$ is the surface Laplacian on the interface of the membrane. The nature of this equation is complex as it is a fourth-order PDE. If