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A CLASSIFICATION OF QUADRATIC HAMILTON-POISSON SYSTEMS IN THREE DIMENSIONS

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Abstract. We classify homogeneous positive semidefinite quadratic Hamilton-Poisson systems on a certain subclass of three-dimensional Lie-Poisson spaces.

1. Introduction

The dual space of a Lie algebra admits a natural Poisson structure, namely the Lie-Poisson structure. Such structures, and more specifically quadratic Hamiltonian systems on these structures, form a natural setting for a variety of dynamical systems. Prevalent examples are Euler's classic equations for the rigid body, its extensions and its generalizations (see, e.g. [15–17, 22, 25, 28, 29]). In particular, a number of Lie-Poisson structures arise naturally in the study of optimal control problems (see e.g. [2–4, 9, 17, 26, 27]). The equivalence of quadratic Hamilton-Poisson systems on Lie-Poisson spaces has been considered only by a few authors ([5, 10, 11, 13, 28, 29]).

In the present paper, we consider quadratic Hamilton-Poisson systems on those three-dimensional Lie-Poisson spaces that admit a global Casimir function. (The spaces that do not admit a global Casimir function exhibit some degeneracies and need to be treated in a somewhat different manner.) Furthermore, we restrict to those systems that are both homogeneous and for which the underlying quadratic form is (positive) semidefinite. Such systems (usually on specific Lie-Poisson spaces) have been considered by several authors ([5–8, 28–30]). We address the equivalence of such systems. A classification (under linear equivalence) is obtained; a complete list of normal forms is exhibited. This is done in two parts. First we classify systems within the context of each three-dimensional Lie-Poisson