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AN S^1-REDUCTION OF NON-FORMAL STAR PRODUCT

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Abstract. Starting from the Moyal product on eight-dimensional canonical Euclidean phase space $T^*\mathbb{R}^4$ with an S^1-symplectic action, we construct a non-formal star product, i.e., the deformation parameter is a real number, on the cotangent bundle of three-dimensional Euclidean space except the origin $T^*(\mathbb{R}^3 \backslash \{0\})$ which is the reduced symplectic manifold by the S^1-action.

1. Introduction

In this paper we construct a non-formal star product on the phase space of the MIC-Kepler problem $T^*(\mathbb{R}^3 \backslash \{0\})$. The phase space is equipped with a symplectic structure given as a sum of the canonical symplectic structure and the closed two form of the configuration space which represents the Dirac's magnetic monopole. The product is given by an S^1 reduction from the Moyal product on the cotangent bundle $T^*\mathbb{R}^4$.

Fedosov [2] gives a general formula for the reduction of star products for the case of formal deformation quantization. However, the situation is quite different when we treat the deformation parameter non formal. Most of techniques used in the formal star product case are not useful to the non-formal star product problems.

MIC-Kepler problem was proposed by McIntosh and Cisneros [9], which is a dynamical system describing a motion of an electron in the hydrogen atom under the influence of Kepler potential and the Dirac's magnetic monopole field.

The MIC-Kepler problem is formulated in terms with S^1 reduction by Iwai-Uwano [3, 4]. On the cotangent bundle $T^*(\mathbb{R}^4 \backslash \{0\})$ the conformal Kepler problem is given and then the MIC-Kepler problem is obtained by the S^1-reduction. The classical system was investigated in [3] and the quantum version was studied in [9–11] where the eigenvalues and the multiplicities were calculated.