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SYMMETRIES OF HAMILTONIAN DYNAMICAL SYSTEMS, MOMENTUM MAPS AND REDUCTIONS

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Abstract. This text presents some basic notions in symplectic geometry, Poisson geometry, Hamiltonian systems, Lie algebras and Lie groups actions on symplectic or Poisson manifolds, momentum maps and their use for the reduction of Hamiltonian systems. It should be accessible to readers with a general knowledge of basic notions in differential geometry. Full proofs of many results are provided.

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