Fifteenth International Conference on
Geometry, Integrability and Quantization
June 7–12, 2013, Varna, Bulgaria
Ivaïlo M. Mladenov, Andrei Ludu
and Akira Yoshioka, Editors
Avangard Prima, Sofia 2014, pp 196–203
doi: 10.7546/giq-15-2014-196-203

THE CLASSICAL MAGNETIZED KEPLER PROBLEMS IN HIGHER ODD DIMENSIONS

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Abstract. The Kepler problem for planetary motion is a two-body dynamic model with an attractive force obeying the inverse square law, and has a direct analogue in any dimension. While the magnetized Kepler problems were discovered in the late 1960s, it is not clear until recently that their higher dimensional analogues can exist at all. Here we present a possible route leading to the discovery of these high dimensional magnetized models.

1. The Kepler Problem and its High Dimensional Analogues

The Kepler problem is the mathematical model for a solar system with a single planet or an atom with a single electron, depending on whether it is considered classically or quantum mechanically. At the classical level, this is a dynamic problem with configuration space $\mathbb{R}^3_* := \mathbb{R}^3 \setminus \{0\}$ and equation of motion

$$\mathbf{r}'' = -\frac{\mathbf{r}}{r^3} \tag{1}$$

where $\mathbf{r}$ is a function of time t taking value in $\mathbb{R}^3_*$, $r = |\mathbf{r}|$ and $\mathbf{r}''$ is the second time-derivative of $\mathbf{r}$. Since the force on the left hand side of equation (1) is a central force, the *angular momentum* $\mathbf{L} := \mathbf{r} \times \mathbf{r}'$ is a constant of motion. A hidden fact is that the *Runge-Lenz vector* $\mathbf{A} := \mathbf{L} \times \mathbf{r}' + \dfrac{\mathbf{r}}{r}$ is a constant of motion. Although the Runge-Lenz vector has been re-discovered several times [3], ironically, neither Carl Runge nor Wilhelm Lenz discovered it.

The intrinsic version of the angular momentum is the two-vector

$$L := \mathbf{r} \wedge \mathbf{r}'$$