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MOTION OF CHARGED PARTICLES IN THE EQUATORIAL PLANE OF A MAGNETIC DIPOLE FIELD

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Abstract. Newton–Lorentz equations describing the motion of charged particles in the equatorial plane of a magnetic dipole field are considered. The parametric equations of the trajectories of the particles are obtained explicitly in terms of Jacobi elliptic functions and elliptic integrals.

1. Introduction

In this article we consider the system of Newton–Lorentz equations describing the planar motion of a charged particle in the equatorial plane of a magnetic dipole field. This system belongs to the class of dynamical systems of two degrees of freedom whose integrability in the Liouville–Arnold sense (see, e.g., [1, Section 5]) has been studied recently by the present authors in [2].

The magnitude of the magnetic dipole field depends only on the distance from the origin and hence, see [3], the corresponding Newton–Lorentz system is integrable by quadratures since it possesses two functionally independent integrals of motion, one of which is the speed of the particle. Here, our aim is using the techniques developed in [2] to express the parametric equations of the trajectories of the particles explicitly in analytic form.