Sixteenth International Conference on
Geometry, Integrability and Quantization
June 6–11, 2014, Varna, Bulgaria
Ivaïlo M. Mladenov, Andrei Ludu
and Akira Yoshioka, Editors
Avangard Prima, Sofia 2015, pp 73–93
doi: 10.7546/giq-16-2015-73-93

QUANTIZED VERSION OF THE THEORY OF AFFINE BODY

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Abstract. In the previous lecture we have introduce and discussed the concept of affinely-rigid, i.e., homogeneously deformable body. Some symmetry problems and possible applications were discussed. We referred also to our motivation by Euler ideas. Below we describe the general principles of the quantization of this theory in the Schrödinger language. The special stress is laid on highly-symmetric, in particular affinely-invariant, models and the Peter-Weyl analysis of wave functions.

MSC: 81P05, 81R05, 20C35, 22E70
Keywords: homogeneously deformable body, Schrödinger quantization, affine invariance, highly symmetric models, Peter-Weyl analysis

1. Introduction

Let us consider quantum-mechanical system in configuration space Q – the n-dimensional differential manifold. In Schrödinger theory pure states are described by complex scalar densities Ψ of weight $1/2$ [13]. The scalar product is given by

$$(\Psi, \Phi) = \int \overline{\Psi}\Phi = \int \overline{\Psi}(q)\Phi(q)\mathrm{d}q^1 \ldots \mathrm{d}q^n.$$

Usually Q is a Riemannian or pseudo-Riemannian space (Q, Γ). Classical kinetic energy is then given by

$$T = \frac{1}{2}\Gamma_{\mu\nu}\frac{\mathrm{d}q^\mu}{\mathrm{d}t}\frac{\mathrm{d}q^\nu}{\mathrm{d}t}.$$

The metric Γ gives rise to the natural volume measure

$$\mathrm{d}\mu_\Gamma(q) = \sqrt{|\det[\Gamma_{\mu\nu}]|}\mathrm{d}q^1 \ldots \mathrm{d}q^n.$$