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CLASSICAL AND QUANTIZATION PROBLEMS IN DEGENERATE AFFINE MOTION

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Abstract. Discussed are classical and quantized models of affinely rigid motion with degenerate dimension, i.e., such ones that the geometric dimensions of the material and physical spaces need not be equal to each other. More precisely, the material space may have dimension lower than the physical space. Physically interesting are special cases $m = 2$ or $m = 1$ and $n = 3$, first of all $m = 2$, $n = 3$, i.e., roughly speaking, the affinely deformable coin in three–dimensional Euclidean space. We introduce some special coordinate systems generalizing the polar and two–polar decompositions in the regular case. This enables us to reduce the dynamics to two degrees of freedom. In quantum case this is the reduction of the Schrödinger equation to multicomponent wave functions of two deformation invariants.

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1. Introduction

The mechanics of affine motion was a subject of plenty of our papers [16–18, 22–24, 26–29]. There are also papers of many other people who developed the subject and discussed various applications and generalizations [3, 5–9, 11, 13–15, 25, 33]. Below we consider the classical and quantum generalizations to dimensionally degenerate models, i.e., such ones that the material space may be lower–dimensional than the physical space. Of course, we have in mind mainly the special case of two–dimensional material space and the three–dimensional physical Euclidean space. Roughly speaking, this is the mechanics of affinely–rigid body moving in the usual