



GENERALIZED KEPLER PROBLEMS AND EUCLIDEAN JORDAN ALGEBRAS

GUOWU MENG

*Department of Mathematics, Hong Kong University of Science and Technology
 Clear Water Bay, Kowloon, Hong Kong*

Abstract. This article is a written version of author’s lecture on generalized Kepler problems at the XVII-th International Conference on Geometry, Integrability and Quantization, June 5-10, 2015 Varna, Bulgaria. It begins with a review of the Kepler problem for planetary motion and its magnetized cousins, from which a surprising relationship with Lorentz transformation emerges. Next, we give a review for euclidean Jordan algebra and the associated universal Kepler problem. Finally, we demonstrate that, via the universal Kepler problem, a suitable Poisson realization of the conformal algebra for a simple euclidean Jordan algebra gives rise to a super integrable model that resembles the Kepler problem. In particular we demonstrate how the Kepler problem and its magnetized cousins are obtained this way.

MSC: 70Hxx, 17Cxx, 17Bxx, 37J35

Keywords: Jordan algebra, Kepler problem, Lorentz transformation, Poisson bracket, principal bundle, super integrable models, symmetric cone

CONTENTS

1. Introduction	73
2. Kepler Problem and its Magnetized Cousins	74
2.1. A New Description for the Non-Colliding Orbits	75
2.2. MICZ-Kepler Problems and Lorentz Transformations	76
3. Euclidean Jordan Algebra	77
3.1. The Formally Real Jordan Algebra Structure on $\mathbb{R}^{1,3}$	77
3.2. The Euclidean Structure on $\Gamma(3)$	78
3.3. Relevance to the Kepler Problem	79
3.4. Definition of Euclidean Jordan algebra	79
3.5. The Classification Theorem of Jordan, von Neumann and Wigner	80