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THE LIE ALGEBRA $\mathfrak{sl}(2,\mathbb{R})$ AND NOETHER POINT SYMMETRIES OF LAGRANGIAN SYSTEMS*

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Abstract. Some aspects of the simple Lie algebra $\mathfrak{sl}(2,\mathbb{R})$ realized as a subalgebra of Noether point symmetries of Lagrangian systems and related inverse problems are discussed, specially in connection to Lagrangians of kinetic type and some geometric properties like sectional curvatures.

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1. Introduction

Emerging originally from the theory of differential equations, the Lie symmetry method, jointly with the pioneering work of E. Noether and subsequent developments, has developed to a standard procedure in the analysis of dynamical systems in classical Lagrangian and Hamiltonian mechanics, allowing a systematized study of the conservation laws of such systems, as well as their integrability and quantization properties [1, 3, 11, 13, 15, 18, 19, 21]. The deep relation to Differential Geometry has been profusely used in applications to General Relativity and cosmology, where the Lie method has provided various important results and allowed classifications of gravitational spaces [2, 8, 9, 20, 25]. While a large number of the approaches is devoted to the analysis of symmetries of systems belonging to a specific (equivalence) class, some inverse-type problems focus either on the geometric or topological properties of orbits, the structure of the constants of the motion or the controllability and stability of dynamical systems [10, 23]. A variant of such inverse problems is given by the analysis of systems admitting a fixed symmetry

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