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GEOMETRIC FLOW APPEARING IN CONSERVATION LAW IN CLASSICAL AND QUANTUM MECHANICS

NAOHISA OGAWA

Hokkaido University of Sciences, Sapporo 006-8585 Japan

Abstract. The appearance of a geometric flow in the conservation law of particle number in classical particle diffusion and in the conservation law of probability in quantum mechanics is discussed in the geometrical environment of a two-dimensional curved surface with thickness ϵ embedded in $\mathbb{E}^3$. In such a system, two-dimensional conservation law needs an additional term just like an anomaly. The additional term can be obtained by the ϵ expansion. This term has a Gaussian and a mean curvature dependence and can be written as the total divergence of geometric flow J^i_G. This fact holds in both classical and quantum mechanics.

MSC: 81S10, 58J65
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1. Introduction

The particle motion on a given curved surface $\mathbb{M}^2$ is an interesting problem in a wide range fields in sciences, for example, diffusion or brownian motion [2, 7, 11–13], chemical biology [1], Josephson effect [6], pattern formation [17], quantum mechanics [3–5, 8–10, 14–16] and so on.

Usually the classical dynamics of particles on such a manifold is expressed just by changing the Laplacian to the Laplace-Beltrami operator in the diffusion equation. However, when the surface has a thickness ϵ, i.e., the configuration space is $\mathbb{M}^2 \times \mathbb{R}^1$, (see Fig.1) the situation is not simple [11].

To make the problem concrete, we first introduce a two-dimensional curved manifold Σ in $\mathbb{E}^3$, and than we consider two similar copies of Σ denoted by Σ' and $\tilde{\Sigma}$, and place them on both sides of Σ at a small distance of $\epsilon/2$.