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LIE-POINT SYMMETRIES PRESERVED BY DERIVATIVE

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Abstract. Conditions to guarantee that a point symmetry $\mathbf{X}$ of an n^{th}-order differential equation $q^{(n)} - \omega = 0$ is simultaneously a point symmetry of its derived equation $q^{(n+1)} - \dot{\omega} = 0$ are analyzed, and the possible types of vector fields established. It is further shown that only the simple Lie algebra $\mathfrak{sl}(2,\mathbb{R})$ for a very specific type of realization in the plane can be inherited by a derived equation.

MSC: 17B66, 34A25, 34A26
Keywords: Exact ODE, point symmetry, simple Lie algebra, symmetry analysis

1. Introduction

The symmetry analysis of (scalar) ordinary differential equations (ODEs in short) that emerged from the pioneering work of Sophus Lie on transformation groups still constitutes, after more than a century, an active field of research with fruitful applications [9, 10, 13, 20]. While the symmetry classification of second-order ODEs and linearizable higher-order equations is well known ([11,12,15] and references therein), an explicit group classification for n^{th}-order equations with $n \geq 3$ is still an open problem. Although the realizations of Lie algebras on the real plane have been known for a long time [7, 13], the impossibility of establishing a full classification of non-semisimple Lie algebras beyond a certain dimension remains the main obstruction to determine unambiguously the various equivalence classes and admissible dimensions of Lie algebras of point symmetries [18,22], and hence to find the canonical forms of ODEs and systems invariant by them.