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A GENERALIZATION OF THE QUANTIZATION OF POISSON MANIFOLDS

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Abstract. We propose a unified perspective of quantization using a categorical approach. From a fixed Poisson algebra, we define quantization categories as subcategories of the R-module category equipped with the structure of classical limits. The generalized quantization categories have a huge structure including matrix regularization, strict deformation quantization, prequantization, Poisson enveloping algebra and so on.

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1. Quantization Category

The main part of this article is a digest of [4], and some new results about general properties of the quantization category are added.

In this section, we review the quantization category [4]. Before we define the quantization category, we have to introduce some categories as a preparation.

Definition 1. *Let M be a fixed Poisson manifold. Let $\mathscr{R}\mathrm{Mod}$ be a category of R-module for a commutative ring R over $\mathbb{C}$. For a Poisson algebra $\mathcal{A}(M)$ on the Poisson manifold M, a subcategory $\mathscr{P}(\mathcal{A}(M))$ of $\mathscr{R}\mathrm{Mod}$ is defined as follows.*

1. $\mathcal{A}(M) \in \mathrm{ob}(\mathscr{P})$.
2. *For arbitrary $M_i \in \mathrm{ob}(\mathscr{P})$, at least a morphism $T_i \in \mathscr{P}(\mathcal{A}(M),\ M_i)$ exists. We call T_i a quantization map.*