

# YANG–MILLS EQUATIONS OF GENERAL CONNECTIONS AND A CERTAIN SOLUTIONS IN THE QUATERNIONIC HOPF FIBRATION OVER FOUR-SPHERE

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**Abstract.** We investigate a version of Yang–Mills theory by means of general connections. In order to deduce a basic equation, which we regard as a version of Yang–Mills equation, we construct a self-action density using the curvature of general connections. The most different point from the usual theory is that the solutions are given in pairs of two general connections. This enables us to get nontrivial solutions as general connections. Especially, in the quaternionic Hopf fibration over four-sphere, we demonstrate that there certainly exist nontrivial solutions, which are made by twisting the well-known BPST anti-instanton.

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## 1. Introduction

Originally general connections was introduced by Otsuki [12, 13] as a unified concept of connections and tensors by means of endomorphisms  $P: TM \rightarrow TM$  of the tangent bundle of the base space  $M$ . There have been a lot of works related to general connections, so called Otsuki connections. For example, Kikuchi [5], Nagayama [9–11], Otsuki [14–18] applied the theory to spacetimes. Metric spaces with regular Otsuki connection were studied by Yildirim and Bektas [19] (Riemann–Otsuki space), Nadj and Moor [8] (Weyl–Otsuki spaces). Abe [1, 2] extended general connections and their curvatures on arbitrary vector bundles by an invariant method. In the previous paper [7], we investigated variational principles of general connections on principal bundles. We defined the notion of action densities,