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# EXTREMALS AND ISOPERIMETRIC EXTREMALS OF THE ROTATIONS IN THE PLANE

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**Abstract.** In the paper we study the extremals and isoperimetric extremals of the rotations in the plane. We found that extremals of the rotations in the plane are arbitrary curves. By studying the Euler-Poisson equations for extended variational problems, we found that the isoperimetric extremals of the rotations in the Euclidian plane are straight lines.

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## 1. Introduction

It is known that the first variational problem appears in Newton (minimal resistance problem in hydrodynamics) followed by the problem solving the mechanical task of the brachistochrone curve. It was firstly solved by Bernoulli in 1698. He also formulated and solved the second variational problem solving the geometrical task of finding the shortest path between two points on the surface, i.e., geodesics. These results published Euler in 1833 and he also developed the general theory of variational calculus. The great contribution to general theory brought Lagrange, Poisson, and Ostrogradski.

In the 19th century, Poincaré studied astronomical three-body gravitation problem leading to the finding of curves, for which geodesic and Gaussian curvature are proportional, see Blaschke [1, p. 229].