

# THE TRANSFORMATION OF COMMUTATIVE PHASE SPACE TO NONCOMMUTATIVE ONE, AND ITS LORENTZ TRANSFORMATION-LIKE FORMS

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**Abstract.** Noncommutative phase space of arbitrary dimension is discussed. We introduce momentum-momentum noncommutativity in addition to coordinate-coordinate noncommutativity. We find an exact form for the linear transformation which relates a noncommutative phase space to the corresponding ordinary one. By using this form, we show that a noncommutative phase space of arbitrary dimension can be represented by the direct sum of two-dimensional noncommutative ones. In two-dimension, we obtain the transformation which relates a noncommutative phase space to commutative one. The transformation has the Lorentz transformation-like forms and can also describe the Bopp's shift.

MSC: 81R60, 81S10

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## 1. Introduction

Previously, it has been suggested that the spacetime may be noncommutative in string theory. The noncommutative geometry appears the low energy effective theory of a D-brane in the background of a Neveu-Schwartz B field [16]. In addition, the noncommutative geometry is considered that can feature spacetime at