Twenty Second International Conference on
Geometry, Integrability and Quantization
June 8–13, 2020, Varna, Bulgaria
Ivaïlo M. Mladenov, Vladimir Pulov
and Akira Yoshioka, Editors
Avangard Prima, Sofia 2021, pp 253–262
doi: 10.7546/giq-22-2021-253-262

Geometry, Integrability and Quantization XXII

AN INTEGRAL FORMULA FOR A RIEMANNIAN MANIFOLD WITH $k > 2$ SINGULAR DISTRIBUTIONS

VLADIMIR ROVENSKI

Department of Mathematics, University of Haifa, 3498838 Haifa, Israel

Abstract. Mathematicians have shown interest in manifolds endowed with several distributions, e.g., webs composed of different regular foliations and multiply warped products, as well as distributions having variable dimensions (e.g., singular Riemannian foliations). In this paper, we extend our previous study of the mixed scalar curvature of two orthogonal singular distributions for the case of $k > 2$ singular (or regular) pairwise orthogonal distributions, prove an integral formula with this kind of curvature, and illustrate it by characterizing autoparallel singular distributions.

MSC: 53C15; 57R25
Keywords: Riemannian metric, singular distribution, second fundamental form, mixed scalar curvature

1. Introduction

Integral formulae are useful for problems in the geometry of foliations [2, 10, 11]

- characterizing of foliations, whose leaves have a given geometric property
- prescribing the higher mean curvatures of the leaves of a foliation
- minimizing functionals (like volume) for tensor fields on a foliated manifold.

The first known integral formula for a closed Riemannian manifold endowed with a codimension one foliation tells us that the integral mean curvature of the leaves vanishes, see [8]. The second formula in the series of total σ_k's – elementary symmetric functions of principal curvatures of the leaves – says that for a codimension one foliation with a unit normal N to the leaves the total σ_2 is a half of the total Ricci curvature in the N-direction, e.g., [2]: $\int_M (2\,\sigma_2 - \mathrm{Ric}_{N,N})\,\mathrm{d}\,\mathrm{vol} = 0$. The above integral formula implies nonexistence of totally umbilical foliations on a