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$4+1$-MOULTON CONFIGURATION AND POSITIVE MASS DEFORMATION

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Abstract. For given k bodies of collinear central configuration of Newtonian k-body problem, we ask whether one can add another body on the line without changing the configuration and motion of the initial bodies so that the total $k+1$ bodies provide a central configuration.

The case $k = 4$ is analyzed. We study the inverse problem of five bodies and obtain a global explicit formula. Then using the formula we find there are five possible positions of the added body and for each case the mass of the added body is zero. We further consider to deform the position of the added body without changing the positions of the initial four bodies so that the total five bodies are in a state of central configuration and the mass of the added body becomes positive. For each solution above, we find such a deformation of the position of the added body in an explicit manner starting from the solution.

MSC: 70F15, 70F10
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1. Introduction

It is well-known that a solution of Newtonian n-body problem on a line becomes *collinear central configuration*, that is, the ratios of the distances of the bodies from the center of mass are constants while the scaling factor is changing. F. R. Moulton [5] generalized a collinear three-body problem which Euler found in 1767 [2] to obtain the following; for a fixed mass vector $\mathbf{m} = (m_1, \ldots, m_n)$ and a fixed