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DEFORMATION QUANTIZATION IN QUANTUM MECHANICS AND QUANTUM FIELD THEORY

ALLEN C. HIRSHFELD

Fachbereich Physik, Universität Dortmund
44221 Dortmund, Germany

Abstract. We discuss deformation quantization in quantum mechanics and quantum field theory. We begin with a discussion of the mathematical question of deforming the commutative algebra of functions on a manifold into a non-commutative algebra by use of an associative product. We then apply these considerations to the commutative algebra of observables of a classical dynamical system, which may be deformed to the non-commutative algebra of quantum observables. This is the process of deformation quantization, which provides a canonical procedure for finding the measurable quantities of a quantum system. The deformation quantization approach is illustrated, first for the case of a simple harmonic oscillator, then for an oscillator coupled to an external source, and finally for a quantum field theory of scalar bosons, where the well-known formula for the number of quanta emitted by a given external source in terms of the Poisson distribution is reproduced.

The relation of the star product method to the better-known methods involving the representation of observables as linear operators on a Hilbert space, or the representation of expectation values as functional integrals, is analyzed. The final lecture deals with a remarkable formula of Cattaneo and Felder, which relates Kontsevich's star product to an expectation value of a product of functions on a Poisson space, and indicates how this formula may be interpreted.

1. Introduction

One may distinguish three main approaches to understanding quantum mechanics (for a more detailed analysis see Styer *et al* [41]). In chronological order the first is the operator formalism, in which physical states are represented as vectors in a Hilbert space, and observables as linear operators on the states. The measurable quantities are the matrix elements of the operators