

## ON SUPERINTEGRABILITY OF THE MANEV PROBLEM AND ITS REAL HAMILTONIAN FORM

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**Abstract.** We construct Ermanno–Bernoulli type invariants for the Manev model dynamics which may be viewed upon as remnants of the Laplace–Runge–Lenz vector in the Kepler model. If the orbits are bounded these invariants exist only when a certain rationality condition is met and thus we have superintegrability only on a subset of initial values. Manev model's dynamics is demonstrated to be bi-Hamiltonian and a recursion operator is constructed. We analyze real form dynamics of the Manev model and derive that it is always superintegrable. We also discuss the symmetry algebras of the Manev model and its real Hamiltonian form.

### 1. Preliminaries

#### 1.1. The Manev Problem

By **Manev model** [15] we mean here the dynamics given by the Hamiltonian

$$H = \frac{1}{2}(p_x^2 + p_y^2 + p_z^2) - \frac{A}{r} - \frac{B}{r^2} \quad (1)$$

where  $r = \sqrt{x^2 + y^2 + z^2}$ ;  $A$  and  $B$  are assumed arbitrary real constants whose positive values correspond to attractive forces. The model itself as proposed by G. Manev involved specific expression for the constant  $B = \frac{3G}{2c^2}A$  determined by applying Max Planck's more general action–reaction principle. It offers a surprisingly good practical approximation to Einstein's relativistic dynamics – at least at