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GALTON–WATSON TREE AND BRANCHING LOOPS

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Abstract. We define a kind of branching process on the loop space by using the branching mechanism of a loop of string theory.

1. Introduction

In conformal field theory or in string theory [7,17] people look at random applications ψ from a Riemann surface Σ into a Riemannian manifold M endowed with the probability measure:

$$\mathrm{d}\mu(\psi) = Z^{-1}\exp[-I(\psi)]\,\mathrm{d}D(\psi) \tag{1}$$

where $\mathrm{d}D(\psi)$ is the formal Lebesgue measure over the set of maps ψ and $I(\psi)$ is the energy of the map ψ. If Σ has boundaries, let us say exit boundaries which are circles S_i^1 and input boundaries which are circles S_i^2, the amplitude related to the measure (1) should realize a map from $\otimes_{output}H$ into $\otimes_{input}H$ where H is an Hilbert space associated to the loop space [42].

In the case where the manifold is the linear space $\mathbb{R}^n$, (1) is a Gaussian measure, which corresponds to the free field measure. Since in two dimension, the Green kernel associated to the Laplacian has a singularity on the diagonal, the random field lives on random distributions [18]. It is difficult to state what is a distribution with values in a curved manifold, because the notion of distribution is linear.

If $\Sigma = [0,1]\times[0,1]$, there is another process indexed by Σ with values in $\mathbb{R}$, which is the Brownian sheet and which is continuous. $\frac{\partial^2}{\partial s\partial t}\psi$ is the white noise over $[0,1]\times[0,1]$. On Σ, there is a natural order, and it is possible after the work of Cairoli [11] to study the stochastic differential equation in Itô meaning:

$$\delta_{s,t}x_{s,t} = A(x_{s,t})\delta_{s,t}\psi \tag{2}$$

by using martingale theory, where A is a vector field over $\mathbb{R}$. This gives an example of a non-gaussian random field parametrized by the square. In the Gaussian case, this gives the Brownian motion over the path space. Doss and Dozzi [13] have