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INTERGABLE SYSTEMS IN MÖBIUS GEOMETRY

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Abstract. This is an overview of classical and recent results on the geometry of isothermic surfaces and conformally flat hypersurfaces in conformal geometry and their relation with curved flats, a particularly simple type of integrable system.

1. Introduction

This text is an account of a series of five lectures given by the author at the 8th International Conference on Geometry, Integrability and Quantization in Varna, Bulgaria. Many of the discussed results are not new and a very wide range of results is covered, in the form of an overview. They are, or in some cases will be, published elsewhere (some of the material has already been known to the classical geometers) in greater detail and references are included for the interested reader, to facilitate further work. In fact, most of the discussed material is elaborated in the author's book [13], where also more pointers to the relevant literature can be found. Thus this text should be understood as an introduction and advertisement for the discussed problems and methods and, as such, is kept in a rather informal and colloquial style.

One of the main ideas, besides providing appropriate background material, was to show how curved flats, a particularly simple type of integrable system, appears in conformal (Möbius) geometry. Here the term "integrable system" is understood in the sense that the set of partial differential equations describing the geometry can be formulated as a zero curvature condition on a loop of connections by introducing a (spectral) parameter. As a consequence, known techniques from integrable systems theory, such as the finite gap integration scheme, can be applied [9]. However, the author's interest is rather in the associated geometry and more exactly in the geometric interpretation of the obtained spectral family and the relation with other (Bäcklund or Darboux type) transformations of the described geometric objects.