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GEOMETRY AND TOPOLOGY OF COADJOINT ORBITS OF SEMISIMPLE LIE GROUPS

JULIA BERNATSKA and PETRO HOLOD

Department of Physical and Mathematical Sciences, National University of Kyiv-Mohyla Academy, 2 Skovorody Str., 04070 Kyiv, Ukraine

Abstract. Orbits of coadjoint representations of classical compact Lie groups have a lot of applications. They appear in representation theory, geometrical quantization, theory of magnetism, quantum optics, etc. As geometric objects the orbits were the subject of extensive study. However, they remain hard for calculation and application. We propose a simple solution for the following problem: an explicit parametrization of the orbit by means of a generalized stereographic projection, which provide a Kählerian structure on the orbit, and basis two-forms for the cohomology group of the orbit.

1. Introduction

Orbits of coadjoint representations of semisimple Lie groups are an extremely interesting subject. These homogeneous spaces are flag manifolds. Remarkable, that the coadjoint orbits of compact groups are Kählerian manifolds. In 1950s Borel, Bott, Koszul, Hirzebruch et al. investigated the coadjoint orbits as complex homogeneous manifolds. It was proven that each coadjoint orbit of a compact connected Lie group G admits a canonical G-invariant complex structure and the only (within homotopies) G-invariant Kählerian metrics. Furthermore, the coadjoint orbits can be considered as fibre bundles whose bases and fibres are coadjoint orbits themselves.

Coadjoint orbits appear in many spheres of theoretical physics, for instance in representation theory, geometrical quantization, theory of magnetism, quantum optics. They serve as definitional domains in problems connected with nonlinear integrable equations (so called equations of soliton type). Since these equations have a wide application, the remarkable properties of coadjoint orbits interest not only mathematicians but also physicists.