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SEARCH FOR THE GEOMETRODYNAMICAL GAUGE GROUP. HYPOTHESES AND SOME RESULTS

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Abstract. Discussed is the problem of the mutual interaction between spinor and gravitational fields. The special stress is laid on the problem of the proper choice of the gauge group responsible for the spinorial geometrodynamics. According to some standard views this is to be the local, i.e., x-dependent, group $\mathrm{SL}(2, \mathbb{C})$, the covering group of the Lorentz group which rules the internal degrees of freedom of gravitational cotetrad. Our idea is that this group should be replaced by $\mathrm{SU}(2, 2)$, i.e., the covering group of the Lorentz group in four dimensions. This leads to the idea of Klein-Gordon-Dirac equation which in a slightly different context was discovered by Barut and coworkers. The idea seems to explain the strange phenomenon of appearing leptons and quarks in characteristic, mysterious doublets in the electroweak interaction.

1. Introductory Remarks. Four-Component versus Two-Component Spinors in Special Relativity

Even now the concept of spinor is still rather mysterious. Let us begin with what is clean, doubtless and experimentally confirmed. Historically the first thing was the discovery by G. Uhlenbeck and S. Goudsmit that to understand the spectral lines of atoms one had to admit the existence of spin — internal angular momentum of electrons of the surprising magnitude $1/2$ in $\hbar$-units. The idea seemed so surprising and speculative that even prominent physicists like Lorentz and Fermi were strongly if not aggressively against it. Fortunately Ehrenfest and Bohr supported the hypothesis [15]. And the strongest support was experimental one, from atomic spectroscopy. The mathematical understanding came later on from group theory. An essential point is that the group $\mathrm{SU}(2)$ may be identified with the universal covering group of $\mathrm{SO}(3, \mathbb{R})$, orthogonal group in three real dimensions, isomorphic with the group of rotations around some fixed point in the physical Euclidean