Geometry of Pre-Contrast Functions and Non-Conservative Estimating Functions

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Abstract. This paper introduces the notion of pre-contrast functions and studies their geometric properties. A contrast function is an asymmetric squared distance like function. A pre-contrast function corresponds to the differential of a contrast function, however, it is not integrable in general. While a contrast function induces a statistical manifold, a pre-contrast function induces a statistical manifold admitting torsion. As an application of pre-contrast functions, geometry of non-conservative estimating functions in statistics is also studied.

Keywords: information geometry, non-conservative estimating function, statistical manifold admitting torsion, pre-contrast function, non-integrable geometry
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INTRODUCTION

A contrast function is a function on a product manifold $M \times M$, introduced to unify the concepts such as squared distance, Kullback-Leibler divergence, or geometric divergence. Since a contrast function ρ is asymmetric in general, ρ induces a dualistic geometric structure on M. The second order derivatives give a semi-Riemannian metric h, and the third order derivatives give a pair of torsion-free affine connections ∇ and ∇^*. In particular, the triplets (M, ∇, h) and (M, ∇^*, h) form mutually dual statistical manifolds (see [2] and [6]). For this reason, contrast functions play an important role in information geometry [1], which is one of geometric theories for a set of probability distributions.

In this paper, we introduce a non-integrable version of contrast functions, called pre-contrast functions. A pre-contrast function corresponds to the differential of a contrast function, however, it is not integrable in general. A pre-contrast function induces a semi-Riemannian metric h and a pair of mutually dual affine connections ∇ and ∇^*. Although the induced connection ∇^* is torsion-free, ∇ has a non-vanishing torsion tensor from the non-integrability of pre-contrast functions. Hence the induced triplet (M, ∇, h) forms a statistical manifold admitting torsion (see Definition 4). In addition, a pre-contrast function has all the information about the induced geometric structures. Hence we show that forth order derivatives give the curvature tensors.

We remark that the notion of statistical manifold admitting torsion was introduced by Kurose to develop a quantum version of information geometry [3]. In the previous papers [3] and [8], the geometry of quantum state spaces was discussed. We also remark that geometry of statistical manifolds and contrast functions can be discussed in the

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