Linear Weingarten Surfaces in Hyperbolic Three-space

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Abstract. We report some results on surfaces in three-dimensional hyperbolic space from the differential geometric viewpoint. We allow the surfaces some kind of singularities.

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INTRODUCTION

In this paper, we present the already known despriction of linear Weingarten surfaces in the hyperbolic 3-space. Details and proofs are referred to papers in the bibliography. Here we give proofs for some basic propositions (Propositions 1 and 3) since they can not be easily traced in the literature.

Linear Weingarten Immersions

Let (N^3, g) be a 3-dimensional Riemannian manifold. Consider an immersion $f : M^2 \to (N^3, g)$, where M^2 is a 2-manifold. Then f is called a *Weingarten immersion (W-immersion)* if

$$dH \wedge dK = 0$$

where H is the mean curvature function and K is the Gaussian curvature function. Constant Gaussian curvature (CGC) immersions and constant mean curvature (CMC) immersions are trivial examples. Rotational surfaces, or more generally, helicoidal surfaces are also Weingarten because both H and K can be represented as functions of one variable.

An immersion f is called a *linear Weingarten immersion (LW-immersion)* if

$$aH + bK + c = 0 \quad \text{for some} \quad [a : b : c] \in RP^2.$$

LW-immersions form a special class in the family of W-immersions. CGC immersions and CMC immersions are again trivial examples. It should be remarked that any parallel surface of them becomes linear Weingarten. More generally, it is not difficult to prove that given a (L)W-immersion f (which is not necessarily CGC nor CMC), any parallel surface f_t of a (L)W-immersion is again (L)W.

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