



ON SOLITON EQUATIONS WITH \mathbb{Z}_h AND \mathbb{D}_h REDUCTIONS: CONSERVATION LAWS AND GENERATING OPERATORS*

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Abstract. The Lax representations for the soliton equations with \mathbb{Z}_h and \mathbb{D}_h reductions are analyzed. Their recursion operators are shown to possess factorization properties due to the grading in the relevant Lie algebra. We show that with each simple Lie algebra one can relate r fundamental recursion operators Λ_{m_k} and a master recursion operator Λ generating NLEEs of MKdV type and their Hamiltonian hierarchies. The Wronskian relations are formulated and shown to provide the tools to understand the inverse scattering method as a generalized Fourier transform. They are also used to analyze the conservation laws of the above mentioned soliton equations.

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