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ANALYTIC REPRESENTATION OF A CLASS OF AXIALLY SYMMETRIC WILLMORE SURFACES

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Abstract. The surfaces providing local extrema to the so-called Willmore functional, which assigns to each surface its total squared mean curvature, are frequently referred to as the Willmore surfaces. The corresponding Euler-Lagrange equation is usually called Willmore equation. The present work is concerned with a special class of axially symmetric solutions to the Willmore equation, which are solutions of a simpler ordinary differential equation. An analytic representation of the corresponding Willmore surfaces is given in terms of Jacobi elliptic functions and elliptic integrals.

1. Introduction

In 1965, T. Willmore proposed (see references [8,9]) to study the surfaces (widely known nowadays as the Willmore surfaces) that provide extremum to the functional

$$\mathcal{W} = \int_{\mathcal{S}} H^2 \mathrm{d}A \tag{1}$$

(often called the Willmore functional), which assigns to each surface $\mathcal{S}$ its total squared mean curvature H. Here $\mathrm{d}A$ denotes the area element of the surface. The corresponding Euler-Lagrange equation (frequently referred to as the Willmore equation in the current literature) read

$$\Delta H + 2(H^2 - K)H = 0 \tag{2}$$