MASSLESS SPINNING PARTICLES ON THE ANTI-DE SITTER SPACETIME

Stephan De Bièvre[1] and Salah Mehdi[2]

UFR de Mathématiques and
Laboratoire de Physique Théorique et Mathématique
Université Paris VII
2, place Jussieu, F-75251 Paris Cedex 05, France

Abstract

We show that unlike what happens on Minkowski spacetime, massless classical particles on the anti-de Sitter spacetime can have a true spin degree of freedom: their phase spaces are eight-dimensional and their world lines lightlike geodesics.

It is well known that the massless non-zero helicity representations of the Poincaré group do not admit a position operator.[1] This phenomenon has a classical counterpart: the corresponding classical particles are not represented by lightlike geodesics but rather by two-planes moving at the speed of light,[2,3] and interpreted as wave fronts. These particles moreover have six-dimensional phase spaces, reflecting the fact that they do not have a spin degree of freedom. On the other hand, massive particles, as well as massless particles of zero helicity, are described classically by geodesics and for them a position operator does not exist in quantum mechanics. We show here that the special status of the massless non-zero helicity particles disappears on the anti-de Sitter spacetime M_κ.

The classical and quantum description of the massive particles on M_κ as well as their behaviour when the curvature κ is taken to zero was studied in detail in Refs.4-5 (and references therein). Their classical motion follows timelike geodesics. The methods of Ref.4 are easily adapted to the zero-mass, zero-helicity case, and it is then easy to see that they move on lightlike geodesics of M_κ as expected. This leaves us with the equivalent of the zero-mass, non-zero helicity case. We turn to its study here. We will show that the phase space of these particles is eight-dimensional, so that they have a spin degree of freedom. Moreover the particles move on lightlike geodesics in M_κ.

The anti-de Sitter spacetime M_κ is the hyperboloid in $\mathbf{R}^5$ given by:

$$y \cdot y = \eta_{\mu\nu} y^\mu y^\nu = \eta^{\mu\nu} y_\mu y_\nu = (y_1)^2 + (y_2)^2 + (y_3)^2 - (y_4)^2 - (y_5)^2 = -\kappa^{-2}.$$

Here η is the standard symmetric quadratic form of signature $(+,+,+,-,-)$ on $\mathbf{R}^5$, which induces on M_κ a Lorentzian metric of signature $(+,+,+,-)$ in the usual way.

---

[1] e-mail: debievre@mathp7.jussieu.fr

[2] e-mail: mehdi@mathp7.jussieu.fr